PROBLEM 2.1.FAM

GIVEN:
Consider a steady-state, two-dimensional heat flux vector field given by
\[ \mathbf{q} = 3x^2 \mathbf{s}_x + 2xy \mathbf{s}_y. \]
The control volume is centered at \( x = a \) and \( y = b \), with sides \( 2\Delta x \) and \( 2\Delta y \) (Figure Pr.2.1).

SKETCH:
Figure Pr.2.1 shows a control volume centered at \( x = a \) and \( y = b \) with side widths of \( 2\Delta x \) and \( 2\Delta y \).

The depth (along \( z \) direction) is \( w \).

OBJECTIVE:
(a) Using the above expression for \( \mathbf{q} \) show that
\[ \lim_{\Delta V \to 0} \frac{\int_A \mathbf{q} \cdot \mathbf{s}_n \, dA}{\Delta V} = \nabla \cdot \mathbf{q}, \]
where the divergence of the heat flux vector is to be evaluated at \( x = a \) and \( y = b \).
Use a length along \( z \) of \( w \) (this will not appear in the final answers). (Hint: Show that you can obtain the same final answer starting from both sides.)
(b) If the divergence of the heat flux vector is nonzero, what is the physical cause?
(c) In the energy equation (2.1), for this net heat flow (described by this heat flux vector field), is the sum of the volumetric terms on the right, causing the nonzero divergence of \( \mathbf{q} \), a heat source or a heat sink? Also is this a uniform or nonuniform volumetric source or sink? Discuss the behavior of the heat flux field for both positive and negative values of \( x \) and \( y \).

SOLUTION:
(a) To prove the validity of \( \mathbf{q} \) above for the region shown in Figure Pr.2.1, we calculate separately the left-hand side and the right-hand side. For the right-hand side we have
\[ \nabla \cdot \mathbf{q} = \left( s_x \frac{\partial}{\partial x} + s_y \frac{\partial}{\partial y} \right) \cdot (3x^2 \mathbf{s}_x + 2xy \mathbf{s}_y). \]
Performing the dot product we have
\[ \nabla \cdot \mathbf{q} = \frac{\partial (3x^2)}{\partial x} + \frac{\partial (2xy)}{\partial y}, \]
which results in
\[ \nabla \cdot \mathbf{q} = 6x + 2x = 8x. \]
Applying the coordinates of the center of the control volume, we have finally

\[ \nabla \cdot \mathbf{q} \big|_{(x=a, y=b)} = 8a. \]

The left-hand side can be divided into four integrals, one for each of the control surfaces:
(i) Control Surface at \( x = a - \Delta x \):
The heat flux vector across this control surface and the normal vector are

\[ \mathbf{q}_1 = 3(a - \Delta x)^2 \mathbf{s}_x + 2(a - \Delta x)y \mathbf{s}_y \]
\[ \mathbf{s}_{n1} = -\mathbf{s}_x. \]

The dot product between \( \mathbf{q} \) and \( \mathbf{s}_n \) is

\[ \mathbf{q}_1 \cdot \mathbf{s}_{n1} = 3(a - \Delta x)^2 (\mathbf{s}_x \cdot -\mathbf{s}_x) + 2(a - \Delta x)y(\mathbf{s}_y \cdot 0) = -3(a - \Delta x)^2. \]

The net heat flow over this control surface is

\[ Q|_{A_1} = \int_{A_1} (\mathbf{q}_1 \cdot \mathbf{s}_{n1}) \, dA = \int_{b - \Delta y}^{b + \Delta y} -3(a - \Delta x)^2 dyw = -6(a - \Delta x)^2 \Delta y w. \]

(ii) Control Surface at \( x = a + \Delta x \):
The heat flux vector across this control surface and the normal vector are

\[ \mathbf{q}_2 = 3(a + \Delta x)^2 \mathbf{s}_x + 2(a + \Delta x)y \mathbf{s}_y \]
\[ \mathbf{s}_{n2} = \mathbf{s}_x. \]

The net heat flow over this control surface is

\[ Q|_{A_2} = \int_{A_2} (\mathbf{q}_2 \cdot \mathbf{s}_{n2}) \, dA = \int_{b - \Delta y}^{b + \Delta y} 3(a + \Delta x)^2 dyw = 6(a + \Delta x)^2 \Delta y w. \]

(iii) Control Surface at \( y = b - \Delta y \):
The heat flux vector across this control surface and the normal vector are

\[ \mathbf{q}_3 = 3x^2 \mathbf{s}_x + 2x(b - \Delta y) \mathbf{s}_y \]
\[ \mathbf{s}_{n3} = -\mathbf{s}_y. \]

The net heat flow over this control surface is

\[ Q|_{A_3} = \int_{A_3} (\mathbf{q}_3 \cdot \mathbf{s}_{n3}) \, dA = \int_{a - \Delta x}^{a + \Delta x} -2x(b - \Delta y) dw \]
\[ = -2(b - \Delta y) (a + \Delta x)^2 - (a - \Delta x)^2 \frac{w}{2} = -4a \Delta x (b - \Delta y) w. \]

(iv) Control Surface at \( y = b + \Delta y \):
The heat flux vector across this control surface and the normal vector are

\[ \mathbf{q}_4 = 3x^2 \mathbf{s}_x + 2x(b + \Delta y) \mathbf{s}_y \]
\[ \mathbf{s}_{n4} = \mathbf{s}_y. \]

The net heat flow over this control surface is

\[ Q|_{A_4} = \int_{A_4} (\mathbf{q}_4 \cdot \mathbf{s}_{n4}) \, dA = \int_{a - \Delta x}^{a + \Delta x} 2x(b + \Delta y) dw \]
\[ = 2(b + \Delta y) (a + \Delta x)^2 - (a - \Delta x)^2 \frac{w}{2} = 4a \Delta x (b + \Delta y) w. \]

Adding up the heat flow across all the surfaces, we have

\[ Q|_A = Q|_{A_1} + Q|_{A_2} + Q|_{A_3} + Q|_{A_4} \]
\[ = \int_{A_1} \mathbf{q}_1 \cdot \mathbf{s}_{n1} \, dA + \int_{A_2} \mathbf{q}_2 \cdot \mathbf{s}_{n2} \, dA + \int_{A_3} \mathbf{q}_3 \cdot \mathbf{s}_{n3} \, dA + \int_{A_4} \mathbf{q}_4 \cdot \mathbf{s}_{n4} \, dA \]
\[ = [-6(a - \Delta x)^2 \Delta y + 6(a + \Delta x)^2 \Delta y - 4a \Delta x (b - \Delta y) + 4a \Delta x (b + \Delta y)] w \]
\[ = (24a \Delta x \Delta y + 8a \Delta x \Delta y) w = 32a \Delta x \Delta y w. \]
Now, applying the limit
\[
\lim_{\Delta V \to 0} \int_A \mathbf{q} \cdot \mathbf{s}_n \, dA = \lim_{\Delta x, \Delta y \to 0} \frac{32a \Delta x \Delta y w}{(2\Delta x)(2\Delta y)w} = \lim_{\Delta x, \Delta y \to 0} 8a = 8a,
\]
which is identical to the result found before. These are the two methods of determining the divergence of the heat flux vector for a given location in the heat transfer medium.

(b) Since this is a steady-state heat flux vector field (i.e., \(q\) is not a function of time \(t\)), the only reason not to have a divergence-free field would be the presence of a heat generation or sink. In this case, the differential-volume energy equation is
\[
\nabla \cdot \mathbf{q} = \sum_i \dot{s}_i.
\]
The heat generation or sink is caused by the conversion of work or other forms of energy to thermal energy. In the energy equation, these energy conversions are called source terms. The source terms \(\dot{s}_i\) could be due to
(i) conversion from physical or chemical bond to thermal energy
(ii) conversion from electromagnetic to thermal energy
(iii) conversion from mechanical to thermal energy

(c) The divergence of the heat flux vector \(\mathbf{q}\) given above is \(8x\). For \(x > 0\), this is a positive source term indicating a heat generation. For \(x < 0\), the source term becomes negative indicating a heat sink. Also, since the source term is a function of \(x\), it is a nonuniform source term in the \(x\) direction and a uniform source term in the \(y\) direction.

COMMENT:

The application of the divergence operator on the heat flux vector (as in the differential-volume energy equation) results in an expression valid for any position \(x\) and \(y\) while the application of the area-integral (as in the integral-volume energy equation) results on a number which is valid only for that specific point in space \(x = a\) and \(y = b\). The integral form of the energy equation gives an integral or overall energy balance over a specified closed region within the medium, while the differential form is pointwise valid, i.e., is satisfied for any point within the medium.

For the control surfaces parallel to the \(x\) axis, the dot product between the heat flux vector and the surface normal was a function of \(x\) (variable). That required the integration along \(x\). The integration is simplified in the case of a constant heat flux vector normal to the control surface, as obtained for the control surfaces parallel to the \(y\) axis. Although the first case is more general, here we will mainly deal with situations in which the heat flux normal to the control surface is constant along the control surface. This will allow the use of the thermal resistance concept and the construction of thermal resistance network models, as it will be discussed starting in Chapter 3.
PROBLEM 2.4.FUN

GIVEN:
A nitrogen meat freezer uses nitrogen gas from a pressurized liquid nitrogen tank to freeze meat patties as they move carried by a conveyor belt. The nitrogen flows inside a chamber in direct contact with the meat patties, which move in the opposite direction. The heat transfer mechanism between the nitrogen gas and the meat patties is surface convection. Meat patties are to be cooled down from their processing (initial) temperature of $T_i = 10^\circ C$ to the storage (final) temperature of $T_o = -15^\circ C$. Each meat patty has a mass $M = 80$ g, diameter $D = 10$ cm, and thickness $l = 1$ cm. Assume for the meat the thermophysical properties of water, i.e., specific heat in the solid state $c_{p,s} = 1,930$ J/kg-K, specific heat in the liquid state $c_{p,l} = 4,200$ J/kg-K, heat of solidification $\Delta h_{ls} = -3.34 \times 10^5$ J/kg, and freezing temperature $T_{ls} = 0^\circ C$. The average surface-convection heat transfer between the nitrogen and the meat patties is estimated as $q_{ku} = 4,000$ W/m² and the conveyor belt moves with a speed of $u_c = 0.01$ m/s.

OBJECTIVE:
(a) Sketch the temperature variation of a meat patty as it move along the freezing chamber.
(b) Neglecting the heat transfer between the conveyor belt and the meat patties, find the length of the freezing chamber. Use the simplifying assumption that the temperature is uniform within the meat patties. This allows the use of a zeroth-order analysis (lumped-capacitance analysis).

SOLUTION:
(a) The temperature variation of the meat patties as they move along the freezing chamber is given in Figure Pr.2.4.

(b) To calculate the necessary length for the freezing chamber, the cooling process is divided into three regimes (shown in Figure Pr.2.4).
(i) Regime 1: Cooling of Liquid
During this period of time, the meat patties are cooled from their initial temperature down to the solidification temperature. Application of the integral-volume energy equation for a control volume enclosing the meat gives

$$\int_{A_{ku}} q_{ku} \cdot s_n dA = \int_V \left(-\frac{d}{dt} \rho c_p T \right) dV.$$

Assuming that $q_{ku}$ is constant and normal to the surface and that the meat temperature and properties are constant throughout the meat patty (lumped-capacitance analysis), the energy equation becomes

$$q_{ku} A_{ku} = -\rho c_p V \frac{dT}{dt}.$$
Integrating the equation above from \( t = t_i = 0 \) to \( t = t_1 \), for a constant \( q_{ku} \), gives

\[
\int_{t_i}^{t_1} q_{ku}A_{ku}dt = -\int_{T_i}^{T_1} \rho c_p V dT
\]

\[
t_1 = \frac{\rho c_p V (T_i - T_{ls})}{q_{ku}A_{ku}} = \frac{M c_p (T_i - T_{ls})}{q_{ku}A_{ku}}
\]

From the data given \( A_{ku} = \frac{\pi D^2}{4} + \pi Dl = 0.011 \text{ m}^2 \) and \( t_1 = 0.08 (\text{kg}) \times \frac{4.200 (\text{J/kg-K}) \times [10 (\text{C}) - 0 (\text{C})]}{4,000 (\text{W/m}^2) 0.011 (\text{m}^2)} = 76.36 \text{ s} = 1.273 \text{ min}. \)

(ii) Regime 2: Solidification

During this regime the meat patties change phase from liquid to solid. Application of the integral-volume energy equation gives

\[
\int_{A_s} q_{ku} \cdot s_n dA = \int_V \dot{s}_{ls} dV.
\]

Again, assuming that \( q_{ku} \) is uniform and normal to the surface and that the meat properties are constant throughout the meat patty (lumped-capacitance analysis), the energy equation becomes

\[
q_{ku}A_{ku} = \dot{s}_{ls} V;
\]

where the volumetric heat consumption due to phase change \( \dot{s}_{ls} \) is obtained from Table 2.1,

\[
\dot{s}_{ls} = -\dot{n}_{ls} \Delta h_{ls}.
\]

The volumetric solidification rate \( \dot{n}_{ls} (\text{kg/m}^3 \cdot \text{s}) \) is given by

\[
\dot{n}_{ls} = \frac{m}{V(t_2 - t_1)}.
\]

Using the relations above, the energy equation becomes

\[
q_{ku}A_{ku} = -\frac{m \Delta h_{ls}}{(t_2 - t_1)},
\]

and solving for \( t_2 - t_1 \),

\[
t_2 - t_1 = -\frac{m \Delta h_{ls}}{q_{ku}A_{ku}}.
\]

From the values given,

\[
t_2 - t_1 = -\frac{0.08 (\text{kg}) (-3.34 \times 10^5) (\text{J/kg})}{4,000 (\text{W/m}^2) 0.011 (\text{m}^2)} = 607.3 \text{ s} = 10.12 \text{ min}. \]

(iii) Regime 3: Cooling of Solid

During this period of time, the meat patties are cooled from the melting temperature down to the final temperature. Application of the lumped-capacitance analysis for a control volume enclosing the meat results in an equation similar to \( t_1 = \rho c_p V (T_i - T_{ls})/q_{ku}A_{ku} = M c_p (T_i - T_{ls})/q_{ku}A_{ku} \), i.e.,

\[
t_o - t_2 = \frac{\rho c_p V (T_{ls} - T_o)}{q_{ku}A_{ku}} = \frac{m c_p (T_{ls} - T_o)}{q_{ku}A_{ku}}.
\]
From the data given,
\[
t_0 - t_2 = \frac{0.08(\text{kg}) \times 1.930(\text{J/kg-K}) \times [0(\degree\text{C}) - (-15)(\degree\text{C})]}{4,000(\text{W/m}^2) \times 0.011(\text{m}^2)} = 52.64 \text{ s} = 0.8773 \text{ min}.
\]

The total time for cooling of the meat patties is therefore

\[
t_o = t_1 + (t_2 - t_1) + (t_o - t_2) = 1.273 + 10.12 + 0.8773 = 12.27 \text{ min}.
\]

For the velocity of the conveyor belt \(u_c = 0.01 \text{ m/s}\), the total length necessary is

\[
L = u_c t_o = 0.01(\text{m/s}) \times 12.27(\text{min}) \times 60(\text{s/min}) = 7.362 \text{ m}.
\]

**COMMENT:**

Phase change at constant pressure for a pure substance occurs at constant temperature.

The temperature evolution for regimes 1 and 3 are linear because \(q_{ku}\) has been assumed constant (note that all the properties are treated as constants). In practice, the heat loss by surface convection depends on the surface temperature and therefore is not constant with time when this surface temperature is changing. This will be discussed in Chapter 6.

The freezing regime accounts for more than 80 s of the total time, while the cooling of solid accounts for only 7 s of the total time. This is a result of the high heat of solidification exhibited by water and the relatively smaller specific heat capacity of ice compared to liquid water. Liquid water has one of the largest specific heat capacities among the pure substances. The specific heat capacity of substances will be discussed in Chapter 3.
GIVEN:
While the integral-volume energy equation (2.9) assumes a uniform temperature and is applicable to many heat transfer media in which the assumption of negligible internal resistance to heat flow is reasonably justifiable, the differential-volume energy equation (2.1) requires no such assumption and justification. However, (2.1) is a differential equation in space and time and requires an analytical solution. The finite-small volume energy equation (2.13) allows for a middle ground between these two limits and divides the medium into small volumes within each of which a uniform temperature is assumed. For a single such volume (2.9) is recovered and for a very large number of such volumes the results of (2.1) are recovered.

Consider friction heating of a disk-brake rotor, as shown in Figure Pr.2.5. The energy conversion rate is $\dot{S}_{m,F}$. The brake friction pad is in contact, while braking, with only a fraction of the rotor surface (marked by $R$). During quick brakes (i.e., over less than $t = 5 \text{s}$), the heat losses from the rotor can be neglected.

Note that $\dot{S}_{m,F}$ remains the same, while $\Delta V$ changes.

$$\dot{S}_{m,F} = 3 \times 10^4 \text{W}, \ R_o = 18 \text{cm}, \ R_i = 13 \text{cm}, \ l = 1.5 \text{cm}, \ R = 15 \text{cm}, \ \rho c_p = 3.5 \times 10^6 \text{J/m}^3\text{-K}, \ T(t = 0) = 20^\circ \text{C}, \ t = 45.$$

SKETCH:
Figure Pr.2.5 shows the rotor and the area under the pad undergoing friction heating.

![Figure Pr.2.5 A disc-brake rotor heated by friction heating. The region under the brake pad contact is also shown.](image)

OBJECTIVE:
Apply (2.13), with (i) the volume marked as the pad contact region, and (ii) the entire volume in Figure Pr.2.5, and determine the temperature $T$ after $t = 4 \text{s}$ for cases (i) and (ii) and the conditions given above.

Note that the resulting energy equation, which is an ordinary differential equation, can be readily integrated.

SOLUTION:
Starting from (2.13), we have

$$Q|_A = -\frac{d}{dt}(\rho c_p T)\Delta V + \dot{S}_{m,F}$$

$$0 = -\rho c_p \Delta V \frac{dT}{dt} + \dot{S}_{m,F}$$

or by separating the variables, using $T|_{t=0} = T(t = 0)$ and integrating from 0 to $t$, we have

$$\int_{T(t=0)}^{T(t)} dT = \int_{t=0}^{t} \frac{\dot{S}_{m,F}}{\rho c_p \Delta V} dt.$$

$$T(t) - T(t = 0) = \frac{\dot{S}_{m,F}}{\rho c_p \Delta V} (t - 0)$$

or

$$T(t) = T(t = 0) + \frac{\dot{S}_{m,F}}{\rho c_p \Delta V} t.$$
Then using the numerical values we have

\[
T(t = 4 \text{ s}) = 20(\text{°C}) + \frac{3 \times 10^4 \text{(W)}}{3.5 \times 10^6 \text{(J/m}^3\cdot\text{K) \times \Delta V} (\text{m}^3)} \times 4(\text{s})
\]

\[
= 20(\text{°C}) + \frac{3.429 \times 10^{-2} \text{(°C)}}{\Delta V}.
\]

(i) The smaller volume gives

\[
\Delta V = \pi(R_o^2 - R_i^2)l
\]

\[
= \pi(0.18^2 - 0.15^2) \text{(m}^2) \times 0.015(\text{m}) = 4.665 \times 10^{-4} \text{ m}^3
\]

\[
T(t = 4 \text{ s}) = 20(\text{°C}) + 73.50(\text{°C}) = 93.50\text{°C}.
\]

(ii) The larger volume gives

\[
\Delta V = \pi(R_o^2 - R_i^2)l
\]

\[
= \pi(0.18^2 - 0.13^2) \text{(m}^2) \times 0.015(\text{m}) = 7.305 \times 10^{-4} \text{ m}^3
\]

\[
T(t = 4 \text{ s}) = 20(\text{°C}) + 46.94(\text{°C}) = 66.94\text{°C}.
\]

**COMMENT:**

For more accurate results, the radial length as well as the length along \(l\) are divided into small-finite volumes and then heat transfer is allowed between them. This is discussed in Section 3.7.
PROBLEM 2.8.FUN

GIVEN:
In some transient heat transfer (i.e., temperature and heat flux vector changing with time) applications, that portion of the heat transfer medium experiencing such a transient behavior is only a small portion of the medium. An example is the seasonal changes of the air temperature near the earth’s surface, which only penetrates a very short distance, compared to the earth’s radius. Then the medium may be approximated as having an infinite extent in the direction perpendicular to the surface and is referred to a semi-infinite medium. Figure Pr.2.8 shows such a medium for the special case of a sudden change of the surface temperature from the initial (and uniform throughout the semi-infinite medium) temperature of \( T(t = 0) \) to a temperature \( T_s \). Under these conditions, the solution for the heat flux is given by

\[
q_{k,x}(x,t) = \frac{k[T_s - T(t = 0)]}{(\pi \alpha t)^{1/2}} e^{-x^2/4\alpha t},
\]

where \( \alpha = k/\rho c_p \) is called the thermal diffusivity.

\( k = 0.25 \) W/m-K (for nylon), \( \alpha = 1.29 \times 10^{-5} \) m\(^2\)/s (for nylon), \( T_s = 105^\circ C \), \( T(t = 0) = 15^\circ C \), \( x_o = 1.5 \) cm, \( t_o = 30 \) s.

This conduction heat flux changes with time and in space.

SKETCH:
Figure Pr.2.8 shows the semi-infinite slab, the conduction heat flux, and the local energy storage/release.

OBJECTIVE:
(a) Using (2.1), with no energy conversion and conduction as the only heat transfer mechanism, determine the time rate of change of local temperature \( \partial T/\partial t \) at location \( x_o \) and elapsed time \( t_o \).
(b) Determine the location of largest time rate of change (rise) in the temperature and evaluate this for the elapsed time \( t_o \).

SOLUTION:
(a) Starting from (2.1) and for no energy conversion and a one-dimensional (in the \( x \) direction) conduction only, we have

\[
\nabla \cdot q = \left( \frac{\partial}{\partial x} q_{k,x} \right) \cdot (q_{k,x} s_x) = -\rho c_p \frac{\partial T}{\partial t}
\]

or

\[
-\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial}{\partial x} q_{k,x}.
\]

Figure Pr.2.8 A semi-infinite slab with an initial temperature \( T(t = 0) \) has its surface temperature suddenly changed to \( T_s \).
Using the given expression for \( q_{k,x}(x,t) \), we have

\[
\frac{\partial T}{\partial t} = - \left( \frac{k}{\rho c_p} \right) \frac{T_s - T(t=0)}{(\pi \alpha t)^{1/2}} \left( \frac{-2x}{4\alpha t} \right) e^{-\frac{x^2}{4\alpha t}}.
\]

Evaluating this at \( x_o \) and \( t_o \), we have

\[
\frac{\partial T}{\partial t} = \frac{T_s - T(t=0)}{2\pi^{1/2}\alpha^{1/2}t_o^{3/2}}xe^{-\frac{x_o^2}{4\alpha t_o}}.
\]

Using the numerical values, we have

\[
\frac{\partial T}{\partial t} = \frac{(105 - 15)({^\circ}\text{C})}{2\pi^{1/2}(1.29 \times 10^{-5})^{1/2}(\text{m}^2/\text{s})^{1/2}(30)^{3/2}(\text{s})^{3/2}} \times 1.510^{-2}(\text{m})e^{-\frac{(1.5 \times 10^{-2})^2(\text{m})^2}{4 \times 1.29 \times 10^{-7}(\text{m}^2/\text{s}) \times 30(\text{s})}}
\]

\[
= 0.6453(\text{C}/\text{s}) \times e^{-0.1453} = 0.5580{^\circ}\text{C}/\text{s}.
\]

(b) We now differentiate the above expression for \( \partial T/\partial t \), with respect to \( x \), at which we find the location of the largest \( \partial T/\partial t \) occurs. Then by differentiating and using \( t = t_o \), we have

\[
\frac{\partial}{\partial x} \frac{\partial T}{\partial t} = \frac{\partial^2 q_{k,x}}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{T_s - T(t=0)}{2\pi^{1/2}\alpha^{1/2}t_o^{3/2}}e^{-\frac{x^2}{4\alpha t_o}} \right) - \frac{2x^2}{4\alpha t_o} e^{-\frac{x^2}{4\alpha t_o}} = 0.
\]

Then

\[
1 - \frac{2x^2}{4\alpha t_o} = 0
\]

or

\[
x = (2\alpha t_o)^{1/2}.
\]

Now using the numerical values, we have

\[
x = [2 \times 1.29 \times 10^{-5}(\text{m}^2/\text{s}) \times 30(\text{s})]^{1/2}
\]

\[
= 0.02782 \text{ m} = 2.782 \text{ cm}.
\]

From part (a), we have

\[
\frac{\partial T}{\partial t} = 1.1968(\text{C}/\text{s}) \times e^{-0.5} = 0.7259{^\circ}\text{C}/\text{s}.
\]

COMMENT:

Note that as expected, \( \partial T/\partial t = 0 \) at \( x = 0 \) (because \( T_s \) is assumed constant). In Section 3.5.1, we will discuss this transient problem and define the penetration front as the location beyond which the effect of the surface temperature change has not yet penetrated and this distance is given as \( x \equiv \delta_o = 3.6(\alpha t)^{1/2} \), as compared to \( x = 1.414(\alpha t)^{1/2} \) for the location of maximum \( \partial T/\partial t \).