Monte-Carlo Simulation for the Reliability Analysis of Multi-Status Network System Based on Breadth First Search

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Abstract—In general engineering network system, the transformation of the working state, from safety to failure, of unit and system is gradual. And there is an intermediate state between the states of being completely safety and completely failing. On the basis of “safety-intermediate-failure” three-stage working mode, this paper divides it into two-stage mode: “safety-non safety” and “non failure-failure”, and realizes the calculation of connective reliability for network system by traditional disjoint algorithm. The concept of reliability vector is put forward, which includes in three indexes such as probability of safety, probability of intermediate and probability of failure for engineering structures. In order to solve the non-polynomial increase hard problem of reliability calculation for super-large network system with intermediate state, according to the breadth first search technology of graph theory, an algorithm for estimate of the network connectivity is presented to evaluate reliability of network by Monte Carlo simulation. The analysis of an example in a certain network system demonstrates the effectiveness and applicability of the established algorithm.

Keywords—network system; breadth first search; Monte Carlo; intermediate state; reliability vector

I. DEFINITIONS OF THREE WORKING MODES AND RELIABILITY VECTOR OF NETWORK SYSTEM

Network system reliability refers to the probability that network completes the predetermined function in limited conditions and time. If the predetermined function is connectivity of network system, network system connective reliability forms. The reliability analysis of network system is to calculate the probability that sources and terminals keep connected after obtaining the reliability vector of various unit constructions under a certain load.

For the complex multi-source and multi-terminal network system, it is assumed under the conditions as following:

1) The arc unit has three states: the safety, intermediate and failure.
2) The arc units constituting the system have the failure independence.

This paper considers different connecting situations of sources and terminals and three kinds of incompatible working states of network system. Generally, there are two groups of definitions. The first group:

1) Safety state—there is a safe minimum path to all arc units between at least one terminal and sources. The logical relation of various system units can be described by the structure function:

\[
R = \bigcup_{k=1}^{N_1} \bigcup_{j=1}^{N_2} \bigcup_{i=1}^{m_{jk}} A_{ijk} = \bigcup_{k=1}^{N_1} \bigcup_{j=1}^{N_2} \bigcap_{i=1}^{m_{jk}} x_{it} \quad (1)
\]

Where, \(A_{ijk}\) is the minimum path set \(i\) between sources \(j\) and terminal \(k\). \(m_{jk}\) is the number of minimum path sets between sources \(j\) and terminal \(k\). \(N_s\) and \(N_t\) are separately sources number and terminals number. \(x_{it}\) represents the arc unit in minimum path, \(t \leq n-1\), \(n\) represents system nodes number.

2) Failure state — each terminal at least has one cut set with a source. Every arc unit is at the failure state. The logical relation of system units can be described by structure function:

\[
F = \bigcap_{k=1}^{N_1} \bigcup_{j=1}^{N_2} \bigcup_{i=1}^{m_{jk}} C_{ijk} = \bigcap_{k=1}^{N_1} \bigcup_{j=1}^{N_2} \bigcap_{i=1}^{m_{jk}} x_{it} \quad (2)
\]
Where, \( C_{ijk} \) is the minimum path \( i \) between sources \( j \) and terminal \( k \). \( x_{it} \) represents the arc unit in minimum cut set.

3) Intermediate state—other situations except (1) (2).

The reliability analysis responding to three working modes is to evaluate the probability of various working states: reliable probability \( \psi_R = P(R) \), failure probability \( \psi_F = P(F) \), intermediate probability \( \psi_M = 1 - \psi_R - \psi_F \). They constitute reliability vector \( \psi = [\psi_R, \psi_M, \psi_F] \), obviously

\[
\psi_R + \psi_F + \psi_M = 1 \tag{3}
\]

Likewise, the following is the second group of definitions of system working states:

1) Safety state—there exists one safe minimum path to all arc units between each terminal and at least one source.

2) Intermediate state—other situations except (1) (2).

3) Failure state—in the system at least one cut set exists between one terminal and the sources. Each arc unit is at the failure state.

The establishment of three working modes makes the reliability analysis not only consider the intermediary uncertainty, but avoid the complexity of "broad reliability theory". Also this method carries on a more comprehensive estimation to the randomness of network working states, and facilitates the project design and the decision-making.

II. THE ANALYTICAL METHOD OF NETWORK RELIABILITY CONSIDERING INTERMEDIATE STATE

In the network structure functions in equation (1) and (2), each arc unit has three states “safety, intermediate and failure”, so the analysis of system reliability becomes more complex. The characteristics of analysis of reliability in network system with the intermediate state lie in the existence of intermediate probability, which causes \( \psi_R + \psi_F \neq 1 \). Therefore, \( \psi_R \) and \( \psi_F \) in the system must be calculated separately [1]-[4].

The Boolean cubic matrix \( R_{m\times n} \) and \( F_{m\times n} \) represent the logical relation of equation (1) and (2). The Boolean cubic matrix addition operator (\( \oplus \)) and the multiplication operator (\( \otimes \)) can separately complete operations of union and intersection within the minimum paths [5]. The multiplication and addition rules of the Boolean cubic matrix \( A_{m\times n} \) and \( B_{m\times n} \) are shown as the formula (4) (5).

\[
R_{m\times n} = A_{m\times n} \otimes B_{m\times n} = \left[ \begin{array}{c} f_1 \\ \vdots \\ f_k \\ \vdots \\ f_{m_1+m_2} \end{array} \right] \quad 1 \leq k \leq m , m = m_1 + m_2 \tag{4}
\]

\[
F_{m\times n} = A_{m\times n} \otimes B_{m\times n} = \left[ \begin{array}{c} f_1 \\ \vdots \\ f_k = a_i \cap b_j \\ \vdots \\ f_{m_1\times m_2} \end{array} \right] \quad 1 \leq k \leq m , m = m_1 \times m_2 \tag{5}
\]

S3: \( \forall k, l , l \geq k + 1 \), if \( f_k \subseteq f_l \), delete \( f_k \), \( m = m - 1 \).

S4: repeat operation of S2-S3 until deletion cannot be operated.

In the equation (4) (5), \( f_1, f_2, \ldots f_m \) is Boolean cubic of logic functions of \( m \) minimum paths. \( a, b \) are two Boolean cubes. \( m \) is the number of rows in matrix \( F_{m\times n} \), \( n \) is the column number.

The disjoint algorithm of the Boolean cubic matrix(\#) can be seen in [6]. After the disjoint of \( R_{m\times n} \) and \( F_{m\times n} \), the Boolean logic function in \( \# R_{m\times n} \), \( \# F_{m\times n} \) has been independent. The reliability probability of network system is

\[
\psi_S = P(R) = \bigcap_{k=1}^{N_r} \bigcup_{j=1}^{N_i} \bigcap_{x_{ij} \in A_{ij}} P(x_{ij}) \tag{6}
\]

\[
= \sum_{i=1}^{m} P(\text{dis} f_i) = \sum_{i=1}^{m} \prod_{x_{ij} \in \text{dis} f_i} P(x_{ij})
\]

Where, \( m \) is the Boolean cubes number of \( \# R_{m\times n} \) matrix under the condition of calculation of reliability. \( \text{dis} f_i \) is the Boolean cubic in \( \# R_{m\times n} \) matrix, i.e. disjoint minimum path.

By similar methods \( \psi_F \) can be obtained, and \( \psi_M \) can be obtained from (3).
III. MONTE-CARLO ALGORITHM FOR RELIABILITY ANALYSIS OF NETWORK SYSTEM

Monte-Carlo Simulation is used to avoid the complexity of calculation in (1) (2). Its basic thought aims to approximately reproduce the destruction condition of network units by means of the destruction probability, through the massive random simulations. Finally, calculate the frequency of connectivity of sources and terminals, and then replace the precise probability analysis with this approximate frequency.

Reference [7] simulates the destruction condition of network and forms the adjacent matrix of the destruction, finds out the forest which grows from various sources, and determines the connectivity of various terminals and the sources; if the terminal is not in the forest, then this node is at the seriously unreliable state. If the terminal is in the forest, it is at the reliable and intermediate state. How to distinguish the two conditions depends on the color of the network forms. *

The concrete steps can be summarized as following:

1) Write down the adjacent matrix $A$ of the network, $A = [a_{ij}]$ is $n \times n$ order matrix, $n$ is the vertex number in the network graph. If nodes $i$ and $j$ are connected, $a_{ij} = 1$. Otherwise, $a_{ij} = 0$. The adjacent matrix $A$ contains the original topological structural information of the network in the computer.

2) In terms of the evaluation of arc unit probability, determine the reliable probability, intermediate probability, and failure probability $\psi_R$, $\psi_M$, $\psi_F$ of the network edges (units).

3) Reflect the three states probability of arc units in a real number zone . Produce a set of random numbers that are evenly distributed among $[0,1]$ by means of random number generator , and these numbers match the network arcs.

4) Compare the random number $r$ on each edge with the corresponding zone of the probability of three states, then determine the working state of the arc segments. Thus, a simulation of network destruction condition is given.

5) According to one of the simulated destruction conditions from the sample, the adjacent matrix of the destruction $D^*$ of the network forms. $D^*$ is adapted from the adjacent matrix $A$ of the network. If edge $e_{ij}$ is seriously damaged, adapt element $a_{ij}$ for 0. If $e_{ij}$ is at the intermediate state, adapt $a_{ij}$ for $x$.

6) In one simulated network, a directed tree grows from a source. Search outward gradually from the source by means of Breadth First Search. When searching for the node $i$, according to $D^*$, if $a_{ij} = 1$, dye the following node $j$ blue, if $a_{ij} = x$, and $j$ has not been dyed, dye it yellow. If $j$ has been dyed, then make no change. After finishing searching all the linked edges adjacent to $i$, go to the next node; repeat the above steps until the forest growing from all the sources comes into being. Obviously, the final results are: the blue node represents reliable state. The yellow represents the intermediate state between reliable and failure states. The node without color represents seriously unreliable state.

7) Repeat steps in 3)-6), calculate the frequency of connectivity of nodes and sources, i.e. count the times to be dyed and the colors of the nodes in the simulation. Then divide the times to be dyed by the times of simulation. Take the dyeing frequency of the coloring node as the approximate value of the corresponding probability. Count the times of each node to be dyed the same color in the simulation. Then count the times of at least one terminal which is dyed a certain color. Thus the reliable probability, intermediate probability and failure probability of the network system are obtained.

IV. CALCULATING EXAMPLE

Fig. 1 shows the general edge-weighted engineering network. The vertex set is $V = \{v_1, v_2, \ldots, v_9\}$, the edge set is $E = \{e_1, e_2, \ldots, e_{13}\}$. $v_1$, $v_9$ are sources. $v_4$, $v_8$ are terminals. The reliable vector of the edge which is subjected to certain load can be seen in Table I. The reliability of the network system is evaluated. The corresponding calculating results can be seen in Table II, Table III.

![Fig. 1. Chart of directed edge-weighted network with intermediate state](image-url)

<table>
<thead>
<tr>
<th>Arc segments</th>
<th>Reliable probability</th>
<th>Intermediate probability</th>
<th>Failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>e1</td>
<td>0.90</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>e2</td>
<td>0.85</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>e3</td>
<td>0.90</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>e4</td>
<td>0.80</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>e5</td>
<td>0.85</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>e6</td>
<td>0.95</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>e7</td>
<td>0.80</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>e8</td>
<td>0.85</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>e9</td>
<td>0.90</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>e10</td>
<td>0.80</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>e11</td>
<td>0.80</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>e12</td>
<td>0.85</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>e13</td>
<td>0.85</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>
TABLE II. CONNECTIVE RELIABILITY VECTOR OF NODES AND SOURCES

<table>
<thead>
<tr>
<th>Node</th>
<th>Type</th>
<th>Reliability state</th>
<th>Intermediate state</th>
<th>Failure state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Node</td>
<td>0.99002</td>
<td>0.00182</td>
<td>0.00816</td>
</tr>
<tr>
<td>2</td>
<td>Node</td>
<td>0.9471</td>
<td>0.02062</td>
<td>0.03228</td>
</tr>
<tr>
<td>3</td>
<td>Node</td>
<td>0.94874</td>
<td>0.02044</td>
<td>0.03082</td>
</tr>
<tr>
<td>4</td>
<td>Terminal</td>
<td>0.98838</td>
<td>0.00164</td>
<td>0.00999</td>
</tr>
<tr>
<td>5</td>
<td>Node</td>
<td>0.97884</td>
<td>0.0041</td>
<td>0.01706</td>
</tr>
<tr>
<td>6</td>
<td>Node</td>
<td>0.98122</td>
<td>0.00422</td>
<td>0.01456</td>
</tr>
<tr>
<td>7</td>
<td>Source</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Terminal</td>
<td>0.95804</td>
<td>0.01092</td>
<td>0.03104</td>
</tr>
<tr>
<td>9</td>
<td>Source</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE III. CALCULATING RESULTS OF RELIABILITY VECTOR OF THE NETWORK SYSTEM.

<table>
<thead>
<tr>
<th>Working States</th>
<th>Analytical Method</th>
<th>Monte-Carlo Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_R$</td>
<td>$\psi_M$</td>
</tr>
<tr>
<td>Group 1</td>
<td>0.99637</td>
<td>0.00298</td>
</tr>
<tr>
<td>Group 2</td>
<td>0.95370</td>
<td>0.03207</td>
</tr>
</tbody>
</table>

Analysis and discussion:
1) This paper describes and divides the working states of the network system, gives corresponding logical structural function, defines the system reliable vector with intermediate condition, and avoids confusion two concepts—unit reliability and system reliability. The reliable vector of terminals 4, 8 in Table II has substantive disparity with network system reliable vector in Table III. Although both vectors have the intermediate state, they are on different levels—unit level and system level. Therefore, when some literatures evaluate network system reliability by calculating the average connective reliability of all the terminals and sources, extending calculation of unit reliability to that of system reliability, i.e. from the low level to the high level, the situation of network system reliability cannot be reflected [8].

2) For the units and systems with intermediate condition, definitions of their working states have incompatibility, and the calculated system reliability of three states have the normalizing.

3) The reliable vector of network system depends on the divisions of “safety-intermediate-failure” and the construction of corresponding structure function. For different network systems, the division standard is different from each other, and so is the structure function. The computed results have disparity, so the system reliability has the multiplicity, see Table III.

4) The system reliable vectors which are obtained separately by applied analytical method and Monte-Carlo simulation are in consistence in Table III.

V. CONCLUSIONS

This paper has expanded the two working mode “failure-reliable” in reliable theory into “safety-intermediate-failure” three working modes. “Safety” probability and “failure” probability was analyzed by the traditional reliable thought that merges the fuzzy elements into the intermediate state. Thus, this paper proposes Monte Carlo method for analyzing the reliability vector of the network system with intermediate state. It differentiates the influence of the clear elements and fuzzy elements to system and evaluates the reliability of the multi-source and multi-terminal system with intermediate state. So a new effective and practical method is offered for the network system's optimization design and the disaster forecasting. It will be promoted to the optimization design of large-scale complex structure and systems in other industries.

VI. REFERENCES